

APPARATUS AND METHOD TO COMPUTE IN JACOBIAN OF HYPERELLIPTIC  
CURVE DEFINED OVER GALOIS FIELD OF CHARACTERISTIC 2

DESCRIPTION

1. Field of the Invention

The present invention relates to an apparatus and a method for computing the sum of a divisor  $D_1 = \text{g.c.d.}((a_1(x)), (y - b_1(x)))$  and a divisor  $D_2 = \text{g.c.d.}((a_2(x)), (y - b_2(x)))$  on Jacobian of a hyperelliptic curve  $y^2 + y = f(x)$  defined over  $\text{GF}(2^n)$ .

2. Background of the Invention

This application discloses an algorithm suited for performing operation on hardware on Jacobian of a hyperelliptic curve defined over  $\text{GF}(2^n)$ . The following explains prerequisite knowledge required to understand the present invention.

## [1] Hyperelliptic Curve and Divisor

There is a field referred to as  $K$ , and its algebraically closed field is referred to as  $K^-$  ( $K$  with a bar on it). A hyperelliptic curve  $C$  of genus  $g$  over  $K$  is defined by an equation of the form:  $y^2 + h(x)y = f(x)$ . Here,  $h(x)$  is a polynomial of a degree  $g$  at most, and  $f(x)$  is a monic polynomial of degree  $2g+1$ . Here, polynomial  $f$  and  $g$  have coefficients in  $K$  and curve  $C$  have no singular points. Also, when rational point  $P=(x,y)$  is given, its opposite point is defined as  $P^-(x, -y-h(x))$  ( $P^-$  is  $P$  with a bar on it). If  $P$  is infinite-point  $P_\infty$ , it shall be  $P_\infty = P_\infty^-$  ( $P_\infty^-$  is  $P_\infty$  with a bar on it). Hereafter, this application assumes a case of field  $K=GF(2^n)$ ,  $h(x)=1$ .

A divisor  $D$  of  $C$  is a finite formal sum of  $K^-$ -points  $P_1 \dots P_r$  and given by

[Expression 1]

The degree of divisor  $D$  is defined by  $\deg D = \sum m_i$ .

[Expression 2]

[Expression 3]

By defining the sum of divisors of  $C$  as

[Expression 4]

$D(C)$ , a set of the entire divisors of  $C$  forms an additive group which is called a divisor group. The entire divisors of degree 0 form a subgroup which is denoted  $D^0(C)$ . The non zero rational function  $h$  of curve  $C$  has a finite number of zeros and poles,  $\text{div}(h)$  which is a divisor of  $h$  is defined by using zeros and poles of  $h$  in

[Expression 5]

Here,  $P_i$  is a zero of rational function  $h$ ,  $m_i$  is its multiplicity,  $Q_i$  is a pole of rational function  $h$ ,  $n_i$  is multiplicity of poles, and  $\text{ord}_{P_i}(h)$  is an order of rational function  $h$  at point  $P_i$ . A divisor of a non zero rational function is called a principal divisor. A set of entire

principal divisors is called a principal divisor group which is denoted  $D^1(C)$ .

In general, since the number of zeros and the number of poles of a rational function are equal if considered including multiplicity (order), it is  $D^1(C) \subset D^0(C)$ . When two divisors  $D_1$  (Expression 1),  $D_2$  (Expression 2)  $\in D^0(C)$  are given, g.c.d.  $(D_1, D_2)$  of two divisors is defined by  $\sum \min(m_i, n_i) P_i - (\sum \min(m_i, n_i) P_\infty)$ . Also, from the expression, it is apparently  $\text{g.c.d.}(D_1, D_2) \in D^0(C)$ .

## [2] Definition of Jacobian

Jacobian is defined to be the quotient group  $D^0(C)/D^1(C)$  about a group (see "Number Theory 2" by Yoshihiko Yamamoto, Iwanami Shoten (1996)). This is denoted as  $J(C)$ . If  $D_1, D_2 \in D^0(C)$  and  $D_1 - D_2 \in D^1(C)$ ,  $D_1, D_2$  are called linearly equivalent.  $\forall D \in D^0(C)$  can be transformed to divisor  $D_1 (m_i \geq 0)$  which satisfy the following conditions.

[Expression 6]

- (1)  $D_1 \sim D$
- (2) If  $P_i$  appears in  $D_1$ , then the point  $P_i^-$  doesn't appear as one of  $P_j$  ( $j \neq i$ ).
- (3) When  $P_i = P_i^-$ ,  $m_i = 1$  at most.

Such a divisor is called a semi-reduced. An element of a Jacobian is uniquely represented by such a semi-reduced divisor subject to the additional condition that

[Expression 7]

Such a divisor is called a reduced divisor.

Any semi-reduced divisor  $D$  can be uniquely represented by  $D = \text{g.c.d.}((a(x)), (y-b(x)))$ . Here,  $a(x) = \prod_i (x-x_i)^{m_i}$  and  $b(x)$  is the unique polynomial of degree  $< \deg(a)$  satisfying  $b(x_i) = y_i$ . A necessary and sufficient condition for  $D$  to be a reduced divisor is  $\deg a \leq g$ . Hereafter,  $\text{g.c.d.}((a(x)), (y-b(x)))$  is denoted as  $\text{div}(a, b)$  following "Computing in the Jacobian of a

Hyperelliptic Curve," D. G. Cantor, Math. Of Comp, 48, No.177, pp.95-101, (1987). In addition, divisor  $D$  is regarded as a pair of polynomials  $a$  and  $b$  hereafter.

The discrete logarithm problem on  $J(C;GF(2^n))$  is the problem of determining an integer  $m$  such that  $D_1 = mD_2$  for  $D_1, D_2 \in J(C;GF(2^n))$ .

### [3] Security Conditions of Jacobian

The conditions which Jacobian  $J(C;GF(2^n))$  must satisfy in order to construct a secure hyperelliptic curve cryptosystem are as follows according to "Construction and Implementation of a Secure Hyperelliptic Curve CryptoSystem," Yasuyuki Sakai, Yuichi Ishizuka and Kouichi Sakurai, SCIS'98-10.1.B, Jan., 1998, etc.

C1  $\#J(C;GF(2^n))$  is divisible by a large prime number.

C2  $(2^n)^k - 1$ ,  $k < (\log 2^n)^2$  is indivisible by the largest prime factor of  $\#J(C;GF(2^n))$ .

C3  $2g+1 < \log 2^n$

## [4] Algorithm for computing in Jacobian

Addition in Jacobian is, for  $D_1, D_2 \in J(C; GF(2^n))$ , to find a reduced divisor  $D'$  which is a linearly equivalent to  $D_1 + D_2$ . According to the aforementioned article of Cantor and, "Hyperelliptic Curve Cryptosystems," N. Koblitz, Journal of Cryptology, 1, pp.139-150, (1989), an algorithm for addition consists of two procedures. In this procedure 1, for input  $D_1 = \text{div}(a_1, b_1)$  and  $D_2 = \text{div}(a_2, b_2)$ , semi-reduced divisor  $D$  is found, such that  $D_1 + D_2 \sim D$  ( $D = \text{div}(a, b)$ ). In procedure 2, with this  $D$  as input, reduced divisor  $D'$  is found, such that  $D \sim D'$  ( $D' = \text{div}(a', b')$ ,  $\deg b' < \deg a'$ ,  $\deg a' \leq g$ ). These procedures are as follows, if the hyperelliptic curve is  $y^2 + h(x)y = f(x)$ .

Procedure 1

Input  $a_1, b_1 \quad D_1 = \text{div}(a_1, b_1)$

$a_2, b_2 \quad D_2 = \text{div}(a_2, b_2)$

Output  $a, b$

(1)  $s_1(x), s_2(x), s_3(x)$  which satisfy  $d = s_1 a_1 + s_2 a_2 + s_3 (b_1 + b_2 + h)$  are

calculated where a greatest common divisor (GCD) of polynomials  $a_1(x)$ ,  $a_2(x)$ ,  $b_1(x)+b_2(x)+h(x)$  is  $d=d(x)$ .

(2)  $a(x)$ ,  $b(x)$  are calculated based on the following expression.

$$a = a_1 a_2 / d^2$$

$$b = (s_1 a_1 b_2 + s_2 a_2 b_1 + s_3 (b_1 b_2 + f)) / d \bmod a$$

### Procedure 2

Input  $a, b$

Output  $a', b'$  D to D'

(1)  $a'(x)$  and  $b'(x)$  are calculated based on the following expression.

$$a' = (f - hb - b^2) / a$$

$$b' = (-h - b) \bmod a'$$

(2) if  $(\deg a' > g)$  then

$$a = a'$$

$$b = b'$$

goto (1)

else end



In particular, procedure 1 can be simplified as follows in the case of doubling.

Procedure 1

$$a = a_1^2$$

$$b = (b_1^2 + f) \bmod a$$

goto procedure 2 (1)

If it is calculated as is with the above algorithm, there is a drawback that operation of a polynomial with a degree  $2g$  becomes necessary leading to increased computation complexity.

SUMMARY OF THE INVENTION

An object of the present invention is to implement computation in Jacobian with less computation complexity.

Another object of the present invention is to make it

possible to implement computation in Jacobian with a smaller hardware size.

As described in the article quoted above, Koblitz proposed a cryptosystem using the discrete logarithm problem on Jacobian of a hyperelliptic curve of which genus is larger than 1. However, it has been shown by Frey that Koblitz's hyperelliptic Cryptosystem using  $g=2$  curve isn't secure. (see "A Remark Concerning  $m$ -Divisibility and the Discrete Logarithm in the Divisor Class Group of Curves," G. Frey, H. G. Ruck, Math. Of Comp, 62, No.206, pp.865-874, (1994)). As to a curve of which genus is 3 or more, several curves which seem to be secure have been found (see "Construction and Implementation of a Secure Hyperelliptic Curve Cryptosystems," Yasuyuki Sakai, Yuichi Ishizuka and Kouichi Sakurai, SCIS'98-10.1.B, Jan., 1998; "A Hyperelliptic Curve Where Jacobian Becomes Almost Prime on a Finite Field of a Small Characteristic," Izuru Kitamura, SCIS' 98-7.1.A, Jan., 1998; and "Public Key Cryptosystems with Cab Curve (1)," S. Arita, IE ICE ISEC97-54 pp.13-23 (1997), etc.).

In general, calculations in  $GF(2^n)$  are suited for hardware implementation for the following reasons. (1) Addition and multiplication can be performed at high speed on a relatively small-scale piece of hardware. (2) Square operation can be easily performed. (3) Inverse can be operated at high speed by a method proposed by Ito-Tsujii ("A Fast Algorithm for Computing Multiplicative Inverse in  $GF(2^m)$  Using Normal Bases," T. Itoh, S. Tsujii, Inform. and Comput., vol.83, No.1, pp.171-177, (1989)). Moreover, a hyperelliptic curve cryptosystem is more suited to hardware implementation than an elliptic curve cryptosystem because the ground field to be used can be smaller than that for an elliptic curve cryptosystem, and the above-mentioned calculation for acquiring a greatest common divisor of polynomial in Cantor's algorithm can be efficiently performed by having multiple multipliers run in parallel. Accordingly, in the present invention, the computational complexity and the hardware size are reduced by improving Cantor's algorithm.

Therefore, it has the following characteristics. Namely, an apparatus for computing the sum of a divisor  $D_1 = \text{g.c.d.}((a_1(x)), (y - b_1(x)))$  and a divisor  $D_2 = \text{g.c.d.}((a_2(x)), (y - b_2(x)))$  on Jacobian of a hyperelliptic curve  $y^2 + y = f(x)$  defined over  $\text{GF}(2^n)$  (g.c.d. is defined in above) comprises: means for storing  $a_1(x)$ ,  $a_2(x)$ ,  $b_1(x)$  and  $b_2(x)$ ; and means for calculating  $q(x) = \{s_1(x)(b_1(x) + b_2(x))\} \bmod a_2(x)$  by using  $s_1(x)$  in  $s_1(x)a_1(x) + s_2(x)a_2(x) = 1$  in case of  $\text{GCD}(a_1(x), a_2(x)) = 1$  where GCD denotes a greatest common divisor of polynomials. Thus, a new function  $q(x)$  is introduced so as to reduce the entire computational complexity and the hardware size. While examples where a hyperelliptic curve is  $y^2 + y = x^7$  are described in detail in the embodiments, this  $q(x)$  can be effectively used even in the case that it is other hyperelliptic curves. Moreover, since the group operation is commutative, the same sum can be acquired by using  $q(x)$  obtained by exchanging  $a_1$ ,  $b_1$  and  $s_1$  for  $a_2$ ,  $b_2$  and  $s_2$ . Hereafter, it may be explained by using only one of the pair in order to avoid complication, yet it has the same meaning if exchanged.  $q(x) = \{s_1(x)(b_1(x) + b_2(x))\} \bmod a_2(x)$  can be replaced by

$$q(x) = \{s_2(x)(b_1(x) + b_2(x))\} \bmod a_1(x).$$

Moreover, in the case of  $D_1 = D_2$ , means for storing  $a_1(x)$  and  $b_1(x)$ ; and means for calculating  $q(x) = Q(b_1^2(x) + f(x) \bmod a_1^2(x), a_1(x))$  where  $Q(A, B)$  is a quotient of  $A/B$  are provided. Thus, a separate  $q(x)$  is defined.

An apparatus for calculating  $a'(x)$  and  $b'(x)$  of a reduced divisor  $D' = \text{g.c.d.}((a'(x)), (y - b'(x)))$  which is a linearly equivalent to  $D_1 + D_2$  for a divisor  $D_1 = \text{g.c.d.}(a_1(x), y - b_1(x))$  and a divisor  $D_2 = \text{g.c.d.}((a_2(x)), (y - b_2(x)))$  on Jacobian of a hyperelliptic curve  $y^2 + y = f(x)$  defined over  $\text{GF}(2^n)$  comprises: means for calculating  $q(x) = s_1(x)(b_1(x) + b_2(x)) \bmod a_2(x)$  by using  $s_1(x)$  in  $s_1(x)a_1(x) + s_2(x)a_2(x) = 1$  in case of  $\text{GCD}(a_1(x), a_2(x)) = 1$  where  $\text{GCD}$  denotes a greatest common divisor of polynomials; means for calculating

$$\alpha(x) = Q(q^2(x)a_1(x), a_2(x)) + Q(f(x), a_1(x)a_2(x))$$

(or  $\alpha(x) = Q(q^2(x)a_2(x), a_1(x)) + Q(f(x), a_1(x)a_2(x))$ ) which is rendered a monic polynomial where  $Q(A, B)$  is a quotient of  $A/B$ ; means for calculating  $\beta(x) = (q(x)a_1(x) + b_4(x) + 1) \bmod \alpha(x)$

(or  $\beta(x) = (q(x)a_2(x) + b_2(x) + 1) \bmod \alpha(x)$ ); means for calculating  $a'(x) = Q(f(x) + \beta^2(x), \alpha(x))$ ; and means for calculating  $b'(x) = (\beta(x) + 1) \bmod a'(x)$ .

On the other hand, in the case of  $D_1 = D_2$ , it comprises: means for calculating  $q(x) = Q(b_1^2(x) + f(x) \bmod a_1^2(x), a_1(x))$  where  $Q(A, B)$  is a quotient of  $A/B$ ; means for calculating  $\alpha(x) = q^2(x) + Q(f(x), a_1^2(x))$  which is rendered a monic polynomial; means for calculating  $\beta(x) = (b_1^2(x) + f(x) \bmod a_1^2(x) + 1) \bmod \alpha(x)$ ; means for calculating  $a'(x) = Q(f(x) + \beta^2(x), \alpha(x))$ ; and means for calculating  $b'(x) = (\beta(x) + 1) \bmod a'(x)$ .

While the above is an organization on the precondition of rendering as hardware, it is also possible to transform them to be implemented by a computer program, etc. In that case, the program will be stored on storage media such as a floppy disk and a CD-ROM and other storage devices.

#### BRIEF DESCRIPTION OF THE DRAWINGS

Fig. 1 is a block diagram of the entire present invention.

Fig. 2 is a diagram showing the initial state of register group 1 in implementing the algorithm of the present invention (ordinary addition).

Fig. 3 is a diagram showing the state of Ureg storing the result in process of  $q(x) = s_1(b_1 + b_2) \bmod a_2$ .

Fig. 4 is a diagram showing the state of Ureg storing the result in process of  $q(x) = s_1(b_1 + b_2) \bmod a_2$ .

Fig. 5 is a diagram showing the state of Ureg storing the result in process of  $q(x) = s_1(b_1 + b_2) \bmod a_2$ .

Fig. 6 is a diagram showing the state of Zreg storing the final result of  $q(x)$ .

Fig. 7 is a diagram showing the state of Ureg storing the

result in process of  $a_4(x) = Q(q^2 a_1, a_2) + x + c_2 + e_2$ .

Fig. 8 is a diagram showing the state of Ureg storing the result in process of  $a_4(x) = Q(q^2 a_1, a_2) + x + c_2 + e_2$ .

Fig. 9 is a diagram showing the state of Ureg storing the result in process of  $a_4(x) = Q(q^2 a_1, a_2) + x + c_2 + e_2$ .

Fig. 10 is a diagram showing the state of Ureg storing the result in process of  $a_4(x) = Q(q^2 a_1, a_2) + x + c_2 + e_2$ .

Fig. 11 is a diagram showing the state of Ureg storing the result in process of  $a_4(x) = Q(q^2 a_1, a_2) + x + c_2 + e_2$ .

Fig. 12 is a diagram showing the state of Ureg storing the result in process of  $a_4(x) = Q(q^2 a_1, a_2) + x + c_2 + e_2$ .

Fig. 13 is a diagram showing the state of Xreg storing the final result of  $a_4(x)$  rendered monic.



Fig. 14 is a diagram showing the state of Ureg storing the result in process of  $b_4(x) = (qa_1 + b_1 + 1) \bmod a_4$ .

Fig. 15 is a diagram showing the state of Ureg storing the result in process of  $b_4(x) = (qa_1 + b_1 + 1) \bmod a_4$ .

Fig. 16 is a diagram showing the state of Ureg storing the result in process of  $b_4(x) = (qa_1 + b_1 + 1) \bmod a_4$ .

Fig. 17 is a diagram showing the state of Ureg storing the result in process of  $b_4(x) = (qa_1 + b_1 + 1) \bmod a_4$ .

Fig. 18 is a diagram showing the state of Ureg storing the result in process of  $b_4(x) = (qa_1 + b_1 + 1) \bmod a_4$ .

Fig. 19 is a diagram showing the state of Yreg and Zreg storing the final result of  $b_4(x)$ .

Fig. 20 is a diagram showing the state of Ureg storing the result in process of  $a_5(x) = Q(x^7 + b_4^2, a_4)$ .

Fig. 21 is a diagram showing the state of Ureg storing the result in process of  $a_5(x) = Q(x^7 + b_4^2, a_4)$ .

Fig. 22 is a diagram showing the state of Ureg storing the result in process of  $a_5(x) = Q(x^7 + b_4^2, a_4)$ .

Fig. 23 is a diagram showing the state of Ureg storing the result in process of  $a_5(x) = Q(x^7 + b_4^2, a_4)$ .

Fig. 24 is a diagram showing the state of Xreg storing the final result of  $a_5(x)$ .

Fig. 25 is a diagram showing the state of Ureg storing the result in process of  $b_5(x) = (b_4 + 1) \bmod a_5(x)$ .

Fig. 26 is a diagram showing the state of Zreg storing the final result of  $b_5(x)$ .

Fig. 27 is a diagram showing the state of Ureg storing the

result in process of  $q(x)=Q(b_3, a_1)$ .

Fig. 28 is a diagram showing the state of Ureg storing the result in process of  $q(x)=Q(b_3, a_1)$ .

Fig. 29 is a diagram showing the state of Ureg storing the final result of  $q(x)$ .

Fig. 30 is a diagram showing the state of Xreg storing  $a_4(x)=q^2(x)+x$  rendered monic.

Fig. 31 is a diagram showing the state of Ureg storing the result in process of  $b_4=(b_3+1) \bmod a_4$ .

Fig. 32 is a diagram showing the state of Ureg storing the result in process of  $b_4=(b_3+1) \bmod a_4$ .

Fig. 33 is a diagram showing the state of Yreg and Zreg storing the final result of  $b_4$ .

Fig. 34 is a flowchart showing the algorithm of the present invention.

Fig. 35 is a diagram showing configuration of an ordinary computer.

#### DESCRIPTION OF THE PREFERRED EMBODIMENTS

First, the basic algorithm of the present invention is explained.

Meanwhile, in performing an addition, a greatest common divisor of polynomials  $a_1$  and  $a_2$  must be acquired. However, when a ground field is large and  $a_1$  and  $a_2$  are the coordinates of two randomly selected elements of the Jacobian, the case  $\text{GCD}(a_1, a_2)=1$  is extremely likely. Therefore, this application deal with the only case of  $\text{GCD}(a_1, a_2)=1$  hereafter, since processing in the case that  $a_1$  and  $a_2$  have no common divisor does not greatly affect performance. In addition, the greatest common divisor of polynomial  $\text{GCD}(a_1, a_2)=1$  is

represented by polynomials  $s_1$  and  $s_2$  as  $s_1(x)a_1(x)+s_2(x)a_2(x)=1$ . Moreover, even in calculating in the case that they have no common divisor, generalized lemmas 1 and 2 for simplifying procedure 1 which is explained later and function  $Q(u,v)$  for simplifying procedure 2 can be used.

Moreover, Euclid's algorithm is usually used for operation of procedure 1 (1) of the background art, namely to acquire polynomials  $s_1$ ,  $s_2$  and a greatest common divisor of polynomials. Euclid's algorithm is used for calculating an error-location polynomial or an error-evaluation polynomial, in decoding of Reed-Solomon code, etc. and it is frequently implemented. For instance, see Japanese Unexamined Patent Publication No. Hei 7-202718 or Japanese Unexamined Patent Publication No. Sho 62-122332. Accordingly, in the present invention, the process for seeking only the greatest common divisor of polynomials  $d$  of the two polynomials and  $s_1$  (or  $s_2$ ) which meets  $d=s_1a_1+s_2a_2$  is handled as already calculated. It can also be calculated by using the following example of implementation (Fig. 1), for instance. Explanation of the

details of actual operation is omitted, but  $s_1(x)$  is output on Yreg. Also,  $s_1(x)$  is normalized so that the common divisor of polynomials  $d$  equals 1.

In the case that  $a_1$  and  $a_2$  have no common divisor, an algorithm for the operation in Jacobian (background art) can be transformed for procedure 1 (2) and thereafter as follows..

Transformation 1 (a case of a normal addition)

Input  $a_1, a_2, b_1, b_2$

Output  $a', b'$

$$a_3(x) = a_1 a_2$$

$$b_3(x) = (s_1 a_1 b_2 + s_2 a_2 b_1) \bmod a_3$$

$$a_4(x) = (f + b_3 + b_3^2) / a_3$$

$a_4(x)$  rendered monic (render the leading coefficient 1)

$$b_4(x) = (b_3 + 1) \bmod a_4(x)$$

while (deg  $a_4(x) > g$ ) {

$$a' = a_5(x) = (f + b_4 + b_4(x)^2) / a_4(x)$$

$$b' = b_5(x) = (b_4+1) \bmod a_5(x)$$

$$a_4(x) = a'$$

$$b_4(x) = b'$$

}end

$f(x)=x^7$  is used for this algorithm. In procedure 2 (1) for an algorithm of the operation, the orders of  $a$  and  $b$  are reduced by 2 and thereafter the orders are reduced by 2 (or 1), which shows that  $a'$  of degree 3 or less is acquired by executing the content of the while loop once. The polynomial operated for remainder in calculation of  $b_3$  is an expression of degree 7, the dividend polynomial for calculation of  $a_4(x)$  is an expression of degree 10, thus requiring plenty of calculation. To reduce it, a new polynomial  $q(x)=s_1(b_1+b_2) \bmod a_2$  is introduced.

#### Lemma 1

The first  $a_4(x)$  in transformation 1 is given by using  $q(x)$  in  $a_4(x)=Q(q^2a_1, a_2)$ . Here,  $Q(u,v)$  is a function which provides a quotient of  $u/v$ .

(Proof)

First, we show  $b_3(x) = qa_1 + b_1$ . Note that above-mentioned assumption,  $s_1a_1 + s_2a_2 = 1$ . And  $\deg a_1a_2 > \deg b_1$ ,  $b_3(x)$  is calculated as follows.

$$\begin{aligned}
 b_3(x) &= (s_1a_1b_2 + s_2a_2b_1) \bmod a_3 \\
 &= (s_1a_1b_2 + (1 + s_1a_1)b_1) \bmod a_1a_2 \\
 &= (s_1a_1(b_1 + b_2)) \bmod a_1a_2 + b_1 \\
 &= \{(s_1(b_1 + b_2)) \bmod a_2\} a_1 + b_1 \\
 &= qa_1 + b_1 \dots \dots \dots (\#)
 \end{aligned}$$

Next, since a division to calculate  $a_4$  is divisible, and it is  $Q(b_3, a_3) = 0$  from  $\deg b_3 < \deg a_3$ ,

$$\begin{aligned}
 a_4(x) &= Q(f + b_3 + b_3^2, a_3) \\
 &= Q(f, a_3) + Q(b_3^2, a_3).
 \end{aligned}$$

If (#) is substituted into the second term, and note  $Q(b_1^2, a_3) = 0$  from  $\deg b_1^2 < \deg a_3$ ,

$$Q(b_3^2, a_3) = Q(q^2a_1^2 + b_1^2, a_3) = Q(q^2a_1^2, a_3)$$

From this,  $a_4(x) = Q(q^2a_1, a_2) + Q(f, a_3)$  Q.E.D.



If input polynomials are defined as

$$a_1(x) = x^3 + c_2x^2 + c_1x + c_0$$

$$a_2(x) = x^3 + e_2x^2 + e_1x + e_0$$

$$b_1(x) = d_2x^2 + d_1x + d_0$$

$$b_2(x) = f_2x^2 + f_1x + f_0,$$

and  $f(x) = x^7$  is used, the second term of  $a_4$  becomes  $Q(x^7, a_3) = x + c_2 + e_2$ . From this, if transformation 1 is rewritten by using  $q(x)$ , it becomes the following algorithm of the present invention.

Algorithm of the present invention (addition)

Input  $a_1, a_2, b_1, b_2$

Output  $a', b'$

$$q(x) = s_1(b_1 + b_2) \bmod a_2$$

$$a_4(x) = Q(q^2a_1, a_0) + x + c_2 + e_2$$

$$a_4(x) \leftarrow a_4(x) / \text{leading coefficient of } a_4(x) \text{ (Monic)}$$

$$b_4(x) = (qa_1 + b_1 + 1) \bmod a_4$$

If  $(\deg a_4 > 3)$  then

$$a' = a_5(x) = Q(x^7 + b_4^2, a_4)$$

```

      b' = b5(x) = (b4+1) mod a5
else  a' = a4, b' = b4
end

```

In the calculation of  $a_5(x)$ ,  $Q(b_4, a_4)=0$  is used because of  $\deg b_4 < \deg a_4$ . In this algorithm,  $a_3(x)$  of degree 10 has disappeared, and it is no longer necessary to calculate remainder polynomial and division by it. Also, multiplication necessary for calculating  $a_4(x)$  is 9 times only, since the degree of  $q^2a_1$  inside  $Q$  is seventh and the ground field has characteristic 2. In addition,  $b_4(x)$  which is not necessary for calculation of  $a_5(x)$  is eliminated from inside  $Q$ . Thus, it becomes possible to significantly reduce the number of calculation.

Next, doubling arithmetic is considered. The following transformation 2 is acquired by transforming procedure 1 (2) of the background art as in the previous case.

#### Transformation 2

Input  $a_1, b_1$

Output  $a', b'$

$$a_3(x) = a_1^2$$

$$b_3(x) = (b_1^2 + f) \bmod a_3$$

$$a_4(x) = (f + b_3 + b_3^2) / a_3$$

$$a_4(x) \leftarrow a_4(x) / \text{leading coefficient of } a_4(x)$$

$$b_4(x) = (b_3 + 1) \bmod a_4$$

while (deg  $a_4 > g$ ) {

$$a' = a_5(x) = (f + b_4 + b_4^2) / a_4$$

$$b' = b_5(x) = (b_4 + 1) \bmod a_5$$

$$a_4 = a', b_4 = b'$$

}

end

As in the case of additions, the degree of the dividend polynomial is tenth for calculation of  $a_4(x)$  and requires plenty of calculation. To reduce it,  $q(x) = Q(b_3, a_1)$  is introduced.

### Lemma 2

$a_4(x)$  in transformation 2 is given by using  $q(x)$  in  $a_4(x) = q^2 + Q(f, a_3)$ .

(Proof)

Since it is  $Q(b_3, a_3) = 0$  from  $\deg b_3 < \deg a_3$ ,

$$a_4(x) = Q(f, a_3) + Q(b_3^2, a_3)$$

Suppose  $b_3 = r_1 + s_1/a_1$   $\deg s_1 < \deg a_1$  ( $r_1 s_1 \in k[x]$ ,  $k$  is a field of characteristic 2), then  $b_3^2 = r_1^2 + s_1^2/a_1^2$  and the second term  $Q(b_3^2, a_3)$  is  $Q(b_3, a_3)^2$ . Therefore,  $a_4(x) = q_2 + Q(f, a_3)$  Q.E.D.

If  $f(x) = x^7$  is used as in the case of additions, it becomes  $Q(x^7, a_3) = x$  since there is no odd-degree term in  $a_3(x) = a_1^2(x)$ . From this, if algorithm 2 is rewritten by  $q(x)$ , the following algorithm of the present invention (doubling arithmetic) is acquired.

Algorithm of the present invention (doubling arithmetic)

Input  $a_1, b_1$

Output  $a', b'$

$$b_3(x) = b_1^2 + x(a_1 - x^3)^2$$

$$q(x) = Q(b_3, a_1)$$

$$a_4(x) = q^2 + Q(f, a_3)$$

$a_4(x) \leftarrow a_4(x) / \text{leading coefficient of } a_4(x)$

$b_4(x) = (b_3+1) \bmod a_4$

if  $(\deg a_4 > g)$  then

$a' = a_5(x) = Q(x^7 + b_4^2, a_4)$

$b' = b_5(x) = (b_4+1) \bmod a_5$

else  $a' = a_4, b' = b_4$

end

Moreover, in the calculation of  $b_3(x)$ , it is used that it becomes  $b_1^2 \bmod a_3 = b_1^2$  from  $x^7 \bmod a_3 = x(x^3)^2 \bmod a_3 = x(a_1 - (a_1 - x^3))^2 \bmod a_3 = x(a_1 - x_3)^2 \bmod a_3 = x(a_1 - x_3)^2$ ,  $\deg b_1^2 < \deg a_3$ . Also, it is not necessary to store a calculation result of  $b_3(x)$ . It is because a square can be implemented on a Galois field of characteristic 2 with small-scale hardware and it is more advantageous to have a squarer than a register in terms of size. In particular, it can be implemented just by bit shift when a normal base is used. When  $b_3(x)$  is necessary,  $a_1(x)$ ,  $b_1(x)$  can be input on a squarer so that its output can be directly used.

As with ordinary additions, in this algorithm,  $a_3(x)$  of degree 10 has disappeared, and it is no longer necessary to calculate remainder polynomial and division by it. Also, the degree of  $a_4(x)$  is fourth and since ground field has characteristic 2, only squaring is necessary for calculating it and not multiplication. In addition, for calculation of  $a_5(x)$ ,  $b_4(x)$  which is not necessary is eliminated from inside  $Q$ .

Meanwhile, lemma 1 holds in cases other than  $h(x)=1$ . Also,  $Q(f, a_3)$  can easily be calculated noting that the degree of  $f$  is  $2g+1$ , and the degree of  $a_3$  is  $2g$ .

In addition, a hyperelliptic curve may be other than  $y^2+y=x^7$  which is used above. For instance, in the case of  $g=3$ , there are  $K=GF(2^{61})$   $f(x)=x^7+x+1$ ,  $K=GF(2^{67})$   $f(x)=x^7+1$ , etc. If the portion of  $x^7$  in the above algorithm is replaced by such  $f(x)$ , it becomes effective to newly introduce  $q(x)$ .

Fig. 1 shows an example of implementation of the above

algorithm. Register group 1 is connected with selector 1 (3) and selector 2 (9). Both selector 1 (3) and selector 2 (9) are connected with multipliers, squaring 5 and inverter 7. Selector 1 (3) is a selector for input to a register, and selector 2 (9) is a selector for input to multipliers, a squaring and an inverter. Moreover, selector 1 (3), selector 2 (9), multipliers, squaring 5 and inverter 7 are controlled by controller 11 as to their operation (indicated by a broken line in Fig. 1). Register group 1 includes registers Ureg, Xreg, Yreg and Zreg, used as a work area and for storing a result, and registers  $a_1$ ,  $a_2$ ,  $b_1$  and  $b_2$  for storing  $a_1(x)$ ,  $a_2(x)$ ,  $b_1(x)$  and  $b_2(x)$  respectively. Moreover, Ureg and Xreg have four locations while the remaining registers have three locations. Furthermore, although it is not illustrated, adders are provided in multipliers, squaring 5, etc. and are operated if additions are instructed by controller 11.

It is explained how the circuit in Fig. 1 operates in implementing algorithm of the present invention ( addition). Fig. 2 shows the initial state of register group 1. As a prerequisite, Yreg is storing each coefficient of

$s_1(x) = s_{12}x^2 + s_{11}x + s_{10}$ . Also,  $a_1$ ,  $a_2$ ,  $b_1$  and  $b_2$  are storing coefficients of  $a_1(x)$ ,  $a_2(x)$ ,  $b_1(x)$  and  $b_2(x)$  respectively. However, the coefficient of the third-order term which is the highest order is 1, so these do not need to be stored. Namely,  $a_1$  is storing  $c_2$ ,  $c_1$  and  $c_0$ ,  $a_2$  is storing  $e_2$ ,  $e_1$  and  $e_0$ ,  $b_1$  is storing  $d_2$ ,  $d_1$  and  $d_0$ , and  $b_2$  is storing  $f_2$ ,  $f_1$  and  $f_0$ .

First, calculation for acquiring  $q(x) = s_1(b_1 + b_2) \bmod a_2$  is performed. Selector 2 (9) fetches necessary values from register group 1 to implement the following calculation and inputs them into multipliers and squaring 5.

$$\begin{aligned} (1) \quad p_4 &= (s_{12}b'_2) \text{ [coefficient of } x^4] \\ p_3 &= (s_{12}b'_1 + s_{11}b'_2) \text{ [coefficient of } x^3] \\ p_2 &= s_{12}b'_0 \text{ [coefficient of } x^2] \end{aligned}$$

Here, it is as follows.

$$\begin{aligned} &(b_1 + b_2) \\ &= (d_2 + f_2)x^2 + (d_1 + f_1)x + (d_0 + f_0) \\ &= b'_2x^2 + b'_1x + b'_0 \end{aligned}$$



Moreover, it is as follows.

$$\begin{aligned}
 & s_1(b_1+b_2) \\
 = & (s_{12}b'_2)x^4 \\
 & +(s_{12}b'_1+s_{11}b'_2)x^3 \\
 & +(s_{12}b'_0+s_{11}b'_1+s_{10}b'_2)x^2 \\
 & +(s_{11}b'_0+s_{10}b'_1)x \\
 & +s_{10}b'_0
 \end{aligned}$$

Accordingly, the calculation of (1) is calculation of perfect coefficients of fourth-order and third-order terms and coefficients of a portion of a second-order term of  $s_1(b_1+b_2)$ . These calculation results are stored in Ureg by selector 1 (3) (Fig. 3: only Ureg is illustrated). Calculation such as (1) is performed because there is a prerequisite that only four of the multipliers and squaring operators 5 can be used at a time, whereas, since Ureg has four registers, it is also possible to calculate coefficients of the top four terms in (1) if the number of multipliers is not limited. Also, since remainder calculation of  $a_2$  is performed, any term of  $s_1(b_1+b_2)$  below third-order which is the highest order of  $a_2$

will remain as is. Accordingly, the result will be the same even if coefficients of second-order or lower terms are added after remainder calculation.

Next, for  $p_4x^4+p_3x^3+p_2x^2$ , remainder calculation of  $a_2(x)=x^3+e_2x^2+e_1x+e_0$  and calculation of  $s_{10}b'_0=p_0$  for a coefficient of a 0-th term of  $s_1(b_1+b_2)$  are performed. Accordingly, selector 2 (9) fetches necessary values and inputs them into multipliers and squaring operators 5.

$$(2) (p_4x^4+p_3x^3+p_2x^2) \bmod a_2$$

$$s_{10}b'_0 \text{ [coefficient of } x^0]$$

If  $(p_4x^4+p_3x^3+p_2x^2) \bmod a_2$  is described further in detail, it will be as follows.

$$p'_3 = (p_3+p_4e_2) \text{ [coefficient of } x^3]$$

$$p'_2 = (p_2+p_4e_1) \text{ [coefficient of } x^2]$$

$$p'_1 = p_4e_0 \text{ [coefficient of } x]$$

And,

$$p'_0 = s_{10}b'_0 \text{ [coefficient of } x^0]$$

is also implemented.

Selector 1 (3) stores these calculation results in Ureg (Fig. 4: only Ureg is illustrated). Calculation such as (2) is performed because the number of multipliers and squaring operators 5 is four.

Next, calculation of  $p'_3x^3+p'_2x^2+p'_1x+p'_0 \bmod a_2$  is performed. If these are described further in detail, it will be as follows, and selector 2 (9) fetches necessary values and inputs them into multipliers and squaring operators 5.

$$(3) \quad p''_2 = (p'_2+p'_3e_2) \text{ [coefficient of } x^2]$$

$$p''_1 = (p'_1+p'_3e_1) \text{ [coefficient of } x^1]$$

$$p''_0 = (p'_0+p'_3e_0) \text{ [coefficient of } x^0]$$

Selector 1 (3) stores these calculation results in Ureg (Fig. 5: only Ureg is illustrated).

Of  $s_1(b_1+b_2)$  in the above calculation,  $(s_{11}b'_1+s_{10}b'_2)x^2+(s_{11}b'_0+s_{11}b'_1)x$  is not considered. Accordingly, to perform the following calculation, selector 2

(9) fetches necessary values and inputs them into multipliers and squaring operators 5.

$$(4) \quad p''_2 + (s_{11}b'_1 + s_{10}b'_2) \quad [\text{coefficient of } x^2]$$

$$p''_1 + (s_{11}b'_0 + s_{10}b'_1) \quad [\text{coefficient of } x]$$

By these calculations,  $q(x) = q_2x^2 + q_1x + q_0$  was acquired. Selector 1 (3) stores these calculation results in Zreg (Fig. 6: only Zreg is illustrated).

Next, calculation of  $a_4(x) = Q(q^2a_1, a_2) + x + c_2 + e_2$  is implemented. For this,  $q^2a_1$  is calculated first. However, it is not necessary to calculate second-order or lower terms, since it is calculation of a quotient of  $a_2$ . To perform the following calculation, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(1) \quad p_7 = q_2^2 \quad [\text{coefficient of } x^7]$$

$$p_6 = q_2^2c_2 \quad [\text{coefficient of } x^6]$$

$$p_5 = q_2^2c_1 + q_1^2 \quad [\text{coefficient of } x^5]$$

$$p_4 = q_1^2c_2 + q_2^2c_0 \quad [\text{coefficient of } x^4]$$

Here, calculation of a third-order term of  $q^2a_1$  is not implemented since the number of multipliers and locations of Ureg are lacking. Selector 1 (3) stores these calculation results in Ureg (Fig. 7: only Ureg is illustrated).

Moreover,  $q^2a_1$  is as follows.

$$\begin{aligned}
 q^2a_1 = & \\
 & q_2^2x^7 + \\
 & q_2^2c_2x^6 + \\
 & (q_2^2c_1+q_1^2)x^5 + \\
 & (q_1^2c_2+q_2^2c_0)x^4 + \\
 & (q_1^2c_1+q_0^2)x^3 + \\
 & (q_1^2c_0+q_0^2c_2)x^2 + \\
 & q_0^2c_1x+q_0^2c_0
 \end{aligned}$$

Along with calculation of (1), calculation of a inversion of  $q_2^2$  is started. For this, selector 2 (9) inputs into inverter 7 a result of  $q_2^2$  calculated by multipliers and squaring operators 5. It is assumed:  $q^- = 1/q_2^2$ .

Next, calculation for acquiring a quotient by  $a_2$  is performed. It is performed by carrying out remainder calculation by  $a_2$ . Accordingly, to perform the following calculation, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$\begin{aligned} (2) \quad p'_6 &= p_6 + p_7 e_2 \text{ [coefficient of } x^6] \\ p'_5 &= p_5 + p_7 e_1 \text{ [coefficient of } x^5] \\ p'_4 &= p_4 + p_7 e_0 \text{ [coefficient of } x^4] \\ p'_3 &= (q_1^2 c_1 + q_0^2) \text{ [coefficient of } x^3] \end{aligned}$$

In calculating a quotient by  $a_2$ ,  $p_7 x^4$  is a term first acquired, and  $p_7$  has already been acquired and  $a_4$  to be finally acquired will be rendered a monic polynomial, not requiring store in Ureg. Selector 1 (3) stores these calculation results in Ureg (Fig. 8: only Ureg is illustrated).

Furthermore, remainder calculation by  $a_2$  is performed.

However, since a coefficient of a third-order term of  $a_4$  (before rendering monic) is also acquired in this calculation, it will be stored in Ureg together. Accordingly, to perform the following calculation, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(3) \quad p''_5 = p'_5 + p'_6 e_2 \quad [\text{coefficient of } x^5]$$

$$p''_4 = p'_4 + p'_6 e_1 \quad [\text{coefficient of } x^4]$$

$$p''_3 = p'_3 + p'_6 e_0 \quad [\text{coefficient of } x^3]$$

Moreover,  $a_4$  before rendering a monic polynomial is described as follows.

$$a_4(x) = a'_{44}x^4 + a'_{43}x^3 + a'_{42}x^2 + a'_{41}x + a'_{40}$$

Here, it is  $a'_{43} = p'_6$  [coefficient of a third-order term of  $a_4$ ].

Selector 1 (3) fetches  $p'_6$  from Ureg and stores it along with these calculation results in Ureg (Fig. 9: only Ureg is illustrated).

Remainder calculation by  $a_2$  is further performed. However,

since a coefficient of a second-order term of  $a_4$  (before rendering monic) is also acquired in this calculation, it will be stored in Ureg together. Accordingly, to perform the following calculation, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(4) \quad p_{34} = p''_4 + p''_5 e_2 \text{ [coefficient of } x^4]$$

$$p_{33} = p''_3 + p''_5 e_1 \text{ [coefficient of } x^3]$$

Here, it is  $a'_{42} = p'_5$  [coefficient of a second-order term of  $a_4$ ].

Selector 1 (3) fetches  $p''_5$  and  $a'_{43}$  from Ureg and stores it along with these calculation results in Ureg (Fig. 10: only Ureg is illustrated).

Remainder calculation by  $a_2$  is further performed. However, since a coefficient of a first-order term of  $a_4$  (before rendering monic) is also acquired in this calculation, it will be stored in Ureg together. Additions of terms other than  $Q$  of  $a_4$  are also performed. Accordingly, to perform the



following calculation, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(5) \ p_{43} = p_{33} + p_{34}e_2 \text{ [coefficient of } x^3]$$

$$a'_{41} = p_{34} + 1 \text{ [coefficient of a first-order term of } a_4]$$

Selector 1 (3) fetches  $a'_{42}$  and  $a'_{43}$  from Ureg and stores them along with these calculation results in Ureg (Fig. 11: only Ureg is illustrated).

Next, to calculate a coefficient of a 0-th-order term of  $a_4$  (before rendering monic) and also to perform additions of terms other than  $Q$  of  $a_4$ , selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(6) \ a'_{40} = p_{43} + c_2 + e_2 \text{ [constant term of } a_4]$$

Selector 1 (3) fetches  $a'_{42}$ ,  $a'_{43}$  and  $a'_{41}$  from Ureg and stores them along with these calculation results in Ureg (Fig. 12: only Ureg is illustrated). Thus, the value of  $a_4$  before rendering monic is acquired.

Next,  $a_4$  is rendered a monic polynomial.  $a_4$  is fourth-order and its coefficient is  $q_2^2$ . Accordingly, awaiting the end of the calculation explained above, each coefficient of Ureg is multiplied by  $q^-$ . Namely, to perform the following calculation, selector 2 (9) fetches necessary values from inverter 7 and register group 1 and inputs them into multipliers and squaring operators 5.

$$(7) a'_{43}q^-$$

$$a'_{42}q^-$$

$$a'_{41}q^-$$

$$a'_{40}q^-$$

Selector 1 (3) stores these calculation results in Xreg (Fig. 13: only Xreg is illustrated). Thus, the value of  $a_4$  rendered a monic polynomial is acquired.

Next,  $b_4(x) = (qa_1 + b_1 + 1) \bmod a_4$  is calculated. First,  $(qa_1 + b_1 + 1)$  is calculated in the following manner because of limitation of the number of Ureg's locations and the number of

multipliers. Moreover, since  $a_4$  is an polynomial of degree 4, the result will be the same even if third or lower terms of  $(qa_1+b_1+1)$  are added after the remainder calculation. Selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(8) \quad p_5 = q_2 \text{ [coefficient of } x^5]$$

$$p_4 = (q_2c_2+q_1) \text{ [coefficient of } x^4]$$

$$p_3 = (q_2c_1+q_1c_2+q_0) \text{ [coefficient of } x^3]$$

$$p_2 = (d_2+q_2c_0) \text{ [coefficient of } x^2]$$

Selector 1 (3) stores these calculation results in Ureg (Fig. 14: only Ureg is illustrated).

Moreover,

$$qa_1+b_1+1 =$$

$$q_2x^5 +$$

$$(q_2c_2+q_1)x^4 +$$

$$(q_2c_1+q_1c_2+q_0)x^3 +$$

$$(d_2+q_2c_0+q_1c_1+q_0c_2)x^2 +$$

$$(d_1+q_1c_0+q_0c_1)x +$$

$$d_0 + q_0 c_0 + 1$$

And the remainder by  $a_4$  is calculated. Moreover, since a term of  $x^1$  appears by this remainder calculation, an additions of  $d_1 x$  is also performed. The following calculation is performed if described in detail. For this, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$\begin{aligned} (9) \quad p'_4 &= p_4 + p_5 a_{43} \text{ [coefficient of } x^4] \\ p'_3 &= p_3 + p_5 a_{42} \text{ [coefficient of } x^3] \\ p'_2 &= p_2 + p_5 a_{41} \text{ [coefficient of } x^2] \\ p'_1 &= p_1 + p_5 a_{40} + d_1 \text{ [coefficient of } x^1] \end{aligned}$$

Selector 1 (3) stores these calculation results in Ureg (Fig. 15: only Ureg is illustrated).

Remainder calculation by  $a_4$  is performed again. Moreover, since a coefficient of term of  $x^0$  is calculated by this remainder calculation, an addition of  $d_0 + 1$  is also performed. The following calculation is performed if described in

detail. For this, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(10) \quad p''_3 = p'_3 + p'_4 a_{43} \quad [\text{coefficient of } x^3]$$

$$p''_2 = p'_2 + p'_4 a_{42} \quad [\text{coefficient of } x^2]$$

$$p''_1 = p'_1 + p'_4 a_{41} \quad [\text{coefficient of } x^1]$$

$$p''_0 = p'_4 a_{40} + d_0 + 1 \quad [\text{coefficient of } x^0]$$

Selector 1 (3) stores these calculation results in Ureg (Fig. 16: only Ureg is illustrated).

Next, in  $(qa_1 + b_1 + 1)$ , the terms which do not influence the remainder calculation of  $a_4$  and have not been added in (8) through (10) are added. The following calculation is performed if described in detail. Selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(11) \quad p_{32} = p''_2 + c_1 q_1 \quad [\text{coefficient of } x^2]$$

$$p_{31} = p''_1 + c_0 q_1 \quad [\text{coefficient of } x^1]$$

Selector 1 (3) stores these calculation results in Ureg (Fig.

17: only Ureg is illustrated).

In  $(qa_1+b_1+1)$ , the terms which do not influence the remainder calculation of  $a_4$  and have not been added in (8) through (11) are added. The following calculation is performed if described in detail. Selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(12) \quad b_{42} = p_{32} + C_2 q_0 \text{ [coefficient of } x^2]$$

$$b_{41} = p_{31} + C_1 q_0 \text{ [coefficient of } x^1]$$

$$b_{40} = p''_0 + C_0 q_0 \text{ [coefficient of } x^0]$$

Selector 1 (3) stores these calculation results in Ureg (Fig.

18: only Ureg is illustrated).

Thus,  $b_4(x)$  is acquired. Moreover, it is denoted as  $b_4(x) = b_{43}x^3 + b_{42}x^2 + b_{41}x + b_{40}$ . Finally, selector 1 (3) stores the contents of Ureg in Yreg and Zreg (Fig. 19: only Yreg and Zreg are illustrated).

Next,  $a_5(x)=Q(x^7+b_4^2, a_4)$  is calculated. Since  $a_4$  is an polynomial of degree 4, the third or lower terms of  $x^7+b_4^2$  are not necessary for calculation of  $Q$ . As it is  $b_4^2=b_{43}^2x^6+b_{42}^2x^4+b_{41}^2x^2+b_{40}^2$ , only  $b_{43}^2x^6+b_{42}^2x^4+x^7$  is used. Namely, to perform the following calculation, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

- (1)  $p_{17} = 1$  [coefficient of  $x^7$ ]
- $p_{16} = b_{43}^2$  [coefficient of  $x^6$ ]
- $p_{15} = 0$  [coefficient of  $x^5$ ]
- $p_{14} = b_{42}^2$  [coefficient of  $x^4$ ]

Selector 1 (3) stores these calculation results in Ureg (Fig. 20: only Ureg is illustrated).

Next, remainder calculation by  $a_4$  is performed. More concretely, the following calculation is performed. Accordingly, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

(2-1)

$$\begin{aligned}
 p_{26} &= p_{16} + p_{17}a_{43} \\
 &= p_{16} + a_{43} \text{ [coefficient of } x^6\text{]} \\
 p_{25} &= p_{15} + p_{17}a_{42} \\
 &= a_{42} \text{ [coefficient of } x^5\text{]} \\
 p_{24} &= p_{14} + p_{17}a_{41} \\
 &= p_{14} + a_{41} \text{ [coefficient of } x^4\text{]}
 \end{aligned}$$

Moreover, it becomes  $a_{53}=p_{17}$  [coefficient of a third-order term of  $a_5$ ].

Selector 1 (3) fetches  $p_{17}=1$  and stores it along with these calculation results in Ureg (Fig. 21: only Ureg is illustrated). Moreover, it will be  $a_5(x)=a_{53}x^3+a_{52}x^2+a_{51}x+a_{50}$ .

Remainder calculation by  $a_4$  is further performed. More concretely, the following calculation is performed. Accordingly, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

(2-2)



$$p_{35} = p_{25} + p_{26}a_{43} \text{ [coefficient of } x^5]$$

$$p_{34} = p_{24} + p_{26}a_{42} \text{ [coefficient of } x^4]$$

Moreover, it becomes  $a_{52} = p_{26}$  [coefficient of a second-order term of  $a_5$ ].

Selector 1 (3) fetches  $p_{17}$  and  $p_{26}$  and stores them along with these calculation results in Ureg (Fig. 22: only Ureg is illustrated).

Remainder calculation by  $a_4$  is further performed. More concretely, the following calculation is performed. Accordingly, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

(2-3)

$$a_{50} = p_{34} + p_{35}a_{43} \text{ [constant term of } a_5]$$

Moreover, it becomes  $a_{51} = p_{35}$  [coefficient of a first-order term of  $a_5$ ].

Selector 1 (3) fetches  $p_{17}$ ,  $p_{26}$  and  $p_{35}$  and stores them along

with these calculation results in Ureg (Fig. 23: only Ureg is illustrated). Thus,  $a_5$  is calculated.

In the process of (3), selector 1 (3) stores  $a_5(x)$  stored in Ureg into Xreg (Fig. 24: only Xreg is illustrated).

Next,  $b_5(x) = (b_4 + 1) \bmod a_5(x)$  is calculated.  $b_4$  is stored in Yreg and Zreg. First, as a process of (4), selector 1 (3) stores  $b_{43}$ ,  $b_{42}$ ,  $b_{41}$  and  $b_{40} + 1$  in Ureg (Fig. 25: only Ureg is illustrated).

Next, remainder calculation by  $a_5$  is performed. The required calculation is described in detail as follows. Accordingly, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

(5)  $b_{52} = b_{42} + b_{43}a_{52}$  [coefficient of a second-order term of  $b_5$ ]

$b_{51} = b_{41} + b_{43}a_{51}$  [coefficient of a first-order term of  $b_5$ ]

$b_{50} = b_{40} + b_{43}a_{50}$  [coefficient of a 0-th-order term of  $b_5$ ]

It is represented as  $b_5(x) = b_{52}x^2 + b_{51}x + b_{50}$ . Selector 1 (3) stores these calculation results in Zreg (Fig. 26: only Ureg is illustrated). Accordingly,  $a_5$  and  $b_5$  are stored in Xreg and Zreg. Moreover, as solution,  $a' = a_5$ ,  $b' = b_5$ .

Operation of the circuit in Fig. 1 in implementing the algorithm of the present invention (doubling arithmetic) is explained. The initial state in Fig. 2 is not so different in the case of doubling arithmetic. However, registers  $a_2$  and  $b_2$  become empty.

First, in order to calculate  $q(x) = Q(b_3, a_1)$ ,  $b_3(x) = b_1^2 + x(a_1 - x^3)^2$  is calculated. However, since  $a_1$  is a polynomial of degree 3 expression, only third-order or higher terms of  $b_3(x)$  need to be calculated. It is as follows if described further in detail. Selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(1) \quad b_{35} = c_2^2 \text{ [coefficient of } x^5]$$

$$b_{34} = d_2^2 \text{ [coefficient of } x^4]$$

$$b_{33} = c_1^2 \text{ [coefficient of } x^3]$$

Selector 1 (3) stores these calculation results in Ureg (Fig. 27: only Ureg is illustrated).

Moreover, it is as follows.

$$\begin{aligned} b_2^1 + x(a_1 - x^3)^2 &= \\ c_2^2 x^5 + d_2^2 x^4 + c_1^2 x^3 + d_1^2 x^2 + c_0^2 x + d_0^2 \\ &= b_{35} x^5 + b_{34} x^4 + b_{33} x^3 + b_{32} x^2 + b_{31} x + b_{30} \end{aligned}$$

Next,  $Q(b_3, a_1)$  is calculated. It is as follows if described further in detail. Selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

(2-1)

$$p_{14} = b_{34} + b_{35} c_2 \text{ [coefficient of } x^4]$$

$$p_{13} = b_{33} + b_{35} c_1 \text{ [coefficient of } x^3]$$

$$p_{12} = b_{35} c_0 \text{ [coefficient of } x^2]$$

Moreover, it is  $q_2 = b_{35}$ . It is represented as  $q(x) = q_2 x^2 + q_1 x + q_0$ .

Selector 1 (3) fetches  $b_{35}$  from register group 1 and stores them along with these calculation results in Ureg (Fig. 28: only Ureg is illustrated).

Likewise, remainder calculation by  $a_1$  is performed. It is as follows if described further in detail. Selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

(2-2)

$$p_{23} = p_{13} + p_{14}c_2 \text{ [coefficient of } x^3]$$

$$(p_{22} = p_{12} + p_{14}c_1 \text{ [coefficient of } x^2])$$

Moreover, it is  $q_1 = p_{14}$ . Also,  $q_0 = p_{23}$ .

Selector 1 (3) fetches  $q_2$  and  $p_{14}$  from register group 1 and stores them along with these calculation results in Ureg (Fig. 29: only Ureg is illustrated).

Moreover, to simultaneously acquire an inversion of  $c_2^2$ , selector 2 (9) receives  $c_2^2$  from squaring operators 5 and

inputs them into inverter 7. Here, it is  $q^- = 1/c_2^2$ .

As it is necessary to render  $a_4(x) = q^2(x) + x$  monic, the following calculation is performed. Selector 2 (9) fetches necessary values from register group 1 and inverter 7 and inputs them into multipliers and squaring operators 5.

(3)  $a_{43} = 0$  [coefficient of  $x^3$ ]

$a_{42} = q_1^2 q^{-2}$  [coefficient of  $x^2$ ]

$a_{41} = 1q^{-2}$  [coefficient of  $x$ ]

$a_{40} = q_0^2 q^{-2}$  [coefficient of  $x^0$ ]

Selector 1 (3) stores these calculation results in Xreg (Fig. 30: only Xreg is illustrated). Moreover, since it is  $a_{44} = 1$  [coefficient of  $x^4$ ], it is not necessary to consciously store it. It is represented as  $a_4(x) = x^4 + a_{43}x^3 + a_{42}x^2 + a_{41}x + a_{40}$ .

Next,  $b_4 = (b_3 + 1) \bmod a_4$  is calculated. Since  $a_4$  is a polynomial of degree 4, calculation results are the same if the third or lower terms of  $(b_3 + 1)$  are added after remainder calculation. Considering limitation of the number of Ureg's

locations and the number of multipliers, the following calculation is performed. Moreover, selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(4) \quad b_{35} = c_2^2 \text{ [coefficient of } x^5]$$

$$b_{34} = d_2^2 \text{ [coefficient of } x^4]$$

$$b_{33} = c_1^2 \text{ [coefficient of } x^3]$$

$$b_{32} = d_1^2 \text{ [coefficient of } x^2]$$

Selector 1 (3) stores these calculation results in Ureg (Fig. 31: only Ureg is illustrated).

And remainder calculation by  $a_4$  is performed. However, a first-order term of  $b_3$  is added. The following calculation is performed if described further in detail. Selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(5) \quad p_{14} = b_{34} + b_{35}a_{43}$$

$$= b_{34} \text{ [coefficient of } x^4]$$

$$p_{13} = b_{33} + b_{35}a_{42} \text{ [coefficient of } x^3]$$

$$p_{12} = b_{32} + b_{35}a_{41} \text{ [coefficient of } x^2]$$

$$p_{11} = b_{35}a_{40} + c_0^2 \text{ [coefficient of } x^1]$$

Selector 1 (3) stores these calculation results in Ureg (Fig. 32: only Ureg is illustrated).

Remainder calculation by  $a_4$  is further performed. However, a constant term of  $b_3$  and 1 are added. The following calculation is performed if described further in detail. Selector 2 (9) fetches necessary values from register group 1 and inputs them into multipliers and squaring operators 5.

$$(6) \quad p_{23} = p_{13} + p_{14}a_{43} \text{ [coefficient of } x^3]$$

$$p_{22} = p_{12} + p_{14}a_{42} \text{ [coefficient of } x^2]$$

$$p_{21} = p_{11} + p_{14}a_{41} \text{ [coefficient of } x^1]$$

$$p_{20} = p_{14}a_{40} + d_0^2 + 1 \text{ [coefficient of } x^0]$$

Selector 1 (3) stores these calculation results in Yreg and Zreg (Fig. 33: only Yreg and Zreg are illustrated). Thus,  $b_4(x)$  is acquired.



Calculation hereafter is the same as ordinary additions.

The above is illustrated as a processing flow as in Fig. 34. First,  $a_1(x)$ ,  $b_1(x)$ ,  $a_2(x)$  and  $b_2$  are input (step 100). In the case of doubling arithmetic, only  $a_1(x)$  and  $b_1(x)$  are input. Next, the process is switched depending on whether it is an ordinary addition or doubling arithmetic (step 110). In the case of doubling arithmetic,  $q^{-1}=1/c_2^2$  is calculated (step 120). Also,  $q(x)=Q(b_1, a_1)$  is stored in Ureg (step 130). In a circuit as in Fig. 1, steps 120 and 130 are simultaneously performed. And then,  $a_4(x)=q^{-2}(Ureg^2+x)$  rendered monic is calculated and stored in Xreg (step 140). On the other hand, if it is determined as an ordinary addition in step 110, a greatest common polynomial of  $a_1$  and  $a_2$  is calculated. If the greatest common polynomial is not 1, it is not handled by the present invention. And  $s_1$  which is  $s_1(x)a_1(x)+s_2(x)a_2(x)=1$  is calculated and stored in Yreg (step 150). Next,  $q(x)=s_1(b_1+b_2) \bmod a_2$  is calculated and stored in Zreg (step 160). And  $q^{-1}=1/q_2^2$  is calculated (step 170). Also,  $Q(q^2a_1, a_2)+x+c_2+e_2$  is calculated and stored in

Ureg (step 180). Steps 170 and 180 are simultaneously performed in a circuit in Fig. 1. And then,  $a_4(x) = q^{-1}Ureg$  rendered monic is calculated and stored in Xreg (step 190).

The following process is in common with an ordinary addition and doubling arithmetic.  $b_4(x) = (b_3 + 1) \bmod a_4$  is calculated and stored in Yreg and Zreg (step 200). However, the definition of  $b_3$  is different depending on whether it is an ordinary addition or doubling arithmetic. And  $a_5(x) = Q(x^7 + b_2^2, a_4)$  is calculated and stored in Xreg (step 210). Finally,  $b_5(x) = (b_4 + 1) \bmod a_5$  is calculated and stored in Zreg (step 220).

A process as in Fig. 34 can be implemented in a computer program for an ordinary computer (Fig. 35 for instance). However, there is a limit to improvement of processing speed since squaring cannot be performed at high speed by an ordinary computer.

Moreover, it is possible to construct an encryptor, a decoder

or an encryption system including them by implementing an apparatus and a program which execute such an algorithm of the present invention.

{Advantages of the Invention}

Operation in Jacobian could successfully be implemented with improved computation complexity.

It was also made possible to implement operation in Jacobian with improved hardware size.

[Evaluation of Computation Complexity]

The number of execution of multiplication of the algorithm (ordinary addition and doubling arithmetic) of the present invention is evaluated. It is defined that hereafter  $m$  means one multiplier performing multiplication once, and  $M$  means multiple multipliers simultaneously performing multiplication once. Namely,  $m$  is used to represent frequency of multiplication and  $M$  represents frequency of multiplier group being executed. Also,  $I$  means computing once for the

multiplicative inverse. Hereafter,  $I$ ,  $M$  and  $m$  are used to represent computation complexity. For instance,  $I+2m$  represents that computing once for the multiplicative inverse and multiplying twice. The following Table 1 and Table 2 summarize computation complexity of an addition and doubling arithmetic.

[Table 1]

Calculation	Computation complexity	Call frequency	Time
GCD	$3I+23m$	$3I+9M$	$3t(I)+9t(M)$
$q(x)$	$15m$	$4M$	$4t(M)$
$a_4(x)$	$I+20m$	$I+6M$	$t(I)+t(M)$
$b_4(x)$	$17m$	$5M$	$5t(M)$
$a_5(x), b_5(x)$	$6m$	$3M$	$3t(M)$
Total	$4I+81m$	$4I+27M$	$4t(I)+22t(M)$

[Table 2]

Calculation	Computation complexity	Call frequency	Time
$q(x)$	$3m$	$2M$	0
$a_4(x)$	$I+2m$	$I+M$	$t(I)+t(M)$
$b_4(x)$	$8m$	$2M$	$2t(M)$
$a_5(x), b_5(x)$	$6m$	$3M$	$3t(M)$
Total	$I+19m$	$I+8M$	$t(I)+6t(M)$

Moreover, in Table 1 and Table 2,  $t(I)$  represents the time for computing the multiplicative inverse and  $t(M)$  represents the time for computing multiplication. Also,  $2^n$  multiplication is disregarded as executable in one clock cycle.

In Table 1 (addition), it is assumed to be  $t(I) > 5t(M)$ . This makes it possible, while computing  $a_4(x)$ , to simultaneously compute the multiplicative inverse for rendering  $a_4(x)$  monic. Furthermore, in Table 2 (doubling arithmetic), it is assumed to be  $t(I) > 2t(M)$ . This makes it possible to concurrently compute  $q(x)$  and compute the multiplicative inverse for rendering  $a_4(x)$  monic.

$t(I)=8t(M)$  holds on  $GF(2^{59})$  by the method described in "A Fast Algorithm for Computing Multiplicative Inverse in  $GF(2^m)$  Using Normal Bases," T. Itoh, S. Tsujii, Inform. and Comput., vol.83, No.1, pp.171-177, (1989) (hereafter referred to as the Itoh- Tsujii method), and if this is used for Table 1 and Table 2, computation complexity is  $113m$  and time is  $54t(M)$  in

the case of an ordinary addition. And computation complexity is  $27m$  and time is  $14t(M)$  in the case of doubling arithmetic. On the other hand, the results obtained from "Construction and Implementation of a Secure Hyperelliptic Curve Cryptosystems," Yasuyuki Sakai, Yuichi Ishizuka and Kouichi Sakurai, SCIS'98-10.1.B, Jan., 1998 (hereafter referred to Reference 1) is as shown in Table 3.

[Table 3]

	Addition		Doubling arithmetic	
	Multiplication	Multiplicative inverse computation	Multiplication	Multiplicative inverse computation
$g=0$	3	1	3	1
$g=3$	401	0	265	0
$g=11$	17477	0	10437	0

If Table 3 is compared with Table 1 and Table 2, the algorithm of the present invention is 3.5 times better in computation complexity and 7 times better in time in the case of an ordinary addition, and 10 times better in computation complexity and 19 times better in time in the case of doubling arithmetic. Also, efficiency of seven multipliers

is 0.572 in the case of ordinary additions, and 0.45 in the case of doubling arithmetic. Accordingly, in the algorithm of the present invention, calculation is performed more efficiently and there is a higher degree of parallelism compared with conventional techniques.

[Evaluation of Processing Performance]

Table 4 shows calculation of time necessary for integer multiplication of 160 bits or so based on Tables 1 and 2. Moreover, it is assumed that doubling arithmetic is performed 160 times and additions 80 times.

[Table 4]

Operating frequency	Clock required for multiplying once		
	Case A $t(M)=59\text{clock}$	Case B $t(M)=8\text{clock}$	Case C $t(M)=1\text{clock}$
20MHz	19.35ms	2.624ms	0.328ms
40MHz	9.68ms	1.312ms	0.164ms
80MHz	4.84ms	0.656ms	0.082ms

On the other hand, in the implementation by software of Reference 1, Alpha 21164 (250MHz) (Alpha is a trademark of Digital Equipment Corp.) was used and processing time

required was 500 $\mu$ s for an addition, 50 $\mu$ s for doubling arithmetic and 118ms for integer multiplication of 160 bits. Compared with this result, hardware implementation of the algorithm of the present invention performs processing, at operating frequency of 20MHz, 5 times faster in Case A, 50 times faster in Case B, and 360 times faster in Case C. Considering that the ratio of processing time for calculation by dedicated hardware to calculation by a general MPU with about 10 times different operating frequency in the RSA cipher is 5 times or so, it can be said that a hyperelliptic curve cryptosystems and the algorithm of the present invention are fairly suited for hardware implementation.

In addition, as regards the 160-bit-key elliptic curve cryptosystem which is considered equal in security, it is reported that it takes time of maximum 3.6ms to sign at operating frequency of 20MHz according to Technical Bulletin, NIKKEI ELECTRONICS, 3/23/1998, (No.712) pp.23, and also that it takes average processing time of 60ms for 27K-gate hardware at operating frequency of 20MHz according to



"Prototyping Hyperelliptic Curve Cryptosystem Chip," Naoya Torii, Souichi Okada, Takayuki Hasebe, Singaku Society Univ., A-7-1, Oct., 1998. Compared with these, the proposed algorithm performs processing equally or several times faster.

Here, an elliptic curve cryptosystem ( $g=1$ ) and a hyperelliptic curve cryptosystem (an arbitrary  $g$  which is  $g > 1$ ) are compared as to processing performance and power consumption. Calculation of a hyperelliptic curve cryptosystem is complicated compared with an elliptic curve cryptosystem. However, Galois field of approximately  $1/g$  can be used. Generally, if a descriptor is  $GF(2^n)$ , hardware volume of a multiplier as well as power consumption is in proportion to the square of  $n$ , and calculation speed is in proportion to  $1/\{1-(\log_n g)\}$ . Accordingly, the dependence of a multiplier's performance on genus is  $g^4\{1+\log_n g+(\log_n g)^2+\dots\}$ . On the other hand, increase in computation complexity is in proportion to  $g^3$ . Thus, asymptotically, a hyperelliptic curve cryptosystem is more

advantageous by  $g\{1+\log_n g+\dots\}$ . Also, from the viewpoint of hardware implementation, it is an advantage that a hyperelliptic curve cryptosystem can implement  $g$ -times parallelism.

[Evaluation in the case of Mapping to a Gate Array]

In the above explanation,  $t(M)$  and the number of multiplication were used for evaluation. To know maximum operating frequency, circuit design must be concretely performed and mapping must be performed to semiconductor technology. So, as to Case B of Table 4 where a multiplier calculates with 8-clock, a case where it was designed by using VHDL (IEEE std 1076-1987) and mapped to CMOS gate array technology (IBM CMOS 5SE) of effective channel length  $L_{eff}=0.27\mu m$  was evaluated. Consequently, the results of maximum delay between registers of 12ns (corresponding to maximum operating frequency of 83MHz) and hardware size of approximately 140K cells were obtained. Each block size is indicated in Table 5.

[Table 5]

Block in Fig. 1	Size (cells)	
Multiplier	34265 cells	7 multipliers
Squaring	1344 cells	3 squaring operators
Inverter	27414 cells	
Register group	18408 cells	
Controller	9749 cells	26 59-bit registers (including 12 coefficients)
Selector 1	37140 cells	
Selector 2	17402 cells	
Total	145722 cells	

Moreover, the total number of 140K cells was implemented by optimizing timing of the total circuit after connecting each block to reduce approximately 5K cells. These operating frequency and size are sufficiently practical numbers compared with encrypted VLSI such as the RSA. Moreover, as a primitive polynomial of  $GF(2^{59})$ ,  $p(x)=x^{59}+x^6+x^5+x^4+x^3+x+1$  was used. The reason is that optimal normal bases (among normal bases of  $GF(2^n)$ , those which can represent a multiplication

result of 1 bit as a sum of  $2n-1$  terms) do not exist in  $GF(2^{59})$ , and a cyclotomic field only exists in an even-numbered extension field when a base field is  $GF(2)$ .